Math 33A Fall 2005 Homework Assignments Eberlein

HW # 7 due Friday October 21

11.3 # 43, 44

11.4 # 1, 3, 13, 15, 25, 27, 33, 34 (no graphing in 33 and 34), 38

Extra Problems

1. In each case find the tangent plane and the linear approximation of $z = f(x,y)$ at the indicated point (x_o, y_o, z_o), where z is defined implicitly by the given equation. Is there any difference ?

a) $x^2 + y^2 - z^2 = 2x\,(y + z) - 2$ $\quad (x_o, y_o, z_o) = (1, 2, 1)$

b) $xyz^2 + 2z^3y + 3xyz = -6$ $\quad (x_o, y_o, z_o) = (1, -1, 1)$

More on quaternions :

2. If $x = 4 + 2i - 3j + 2k$ and $y = -2 + 3i + 4j + 4k$, then compute the product xy.

3. Let P denote the purely imaginary quaternions, those for which the real part is zero, or equivalently, those quaternions x for which $\bar{x} = -x$, where $\bar{x}$ denotes the conjugate of x. Recall that we can identify $\mathbb{R}^3$ with P by identifying $x = (x_1, x_2, x_3)$ with $x_1i + x_2j + x_3k$ in P. Prove that if $a = (a_1, a_2, a_3)$ in $\mathbb{R}^3$ corresponds to $a_1i + a_2j + a_3k$ in P and $b = (b_1, b_2, b_3)$ in $\mathbb{R}^3$ corresponds to $b_1i + b_2j + b_3k$ in P, then a x b (cross product in $\mathbb{R}^3$) corresponds to $(1/2)(ab - ba)$ in P.

4. (Inverses and cancellation)

a) If x is any nonzero quaternion and $y = \bar{x} / |x|^2$, then show that $xy = yx = 1$.

b) Suppose that $xy = xz$ for quaternions x,y,z with x nonzero. Show that $y = z$.

5. Show that if a is a nonzero purely imaginary quaternion, then $a^2 = -|a|^2$, a negative real number. Conversely, show that if a^2 is a negative real number, then a is a purely imaginary quaternion. (Hint : Use one of the equivalent descriptions of P given in problem 3.)