

HW # 6

due Friday October 14

10.4 # 21, 25

11.3 # 13, 14, 15, 35, 36, 47, 48

Extra Problems

1. Let $\mathbf{n} = (a,b,c)$ be a fixed vector in \mathbb{R}^3 . Let $\mathbf{r}(t) = (x(t),y(t),z(t))$ be a curve in \mathbb{R}^3 such that $\mathbf{r}(t) \cdot \mathbf{n} = d$ (dot product) for all t , where d is some real number. Show that $\mathbf{r}(t)$ lies in the plane with normal \mathbf{n} that contains the point $\mathbf{r}(0)$.

2. Prove that $\mathbf{T}(t) \times \mathbf{N}(t) = \mathbf{r}'(t) \times \mathbf{r}''(t) / |\mathbf{r}'(t) \times \mathbf{r}''(t)|$ for all t .

(Hint : Use the following formulas from class or from pp. 720-721 of the textbook :

$\mathbf{r}'(t) = v \mathbf{T}(t)$ and $\mathbf{r}''(t) = \frac{dv}{dt} \mathbf{T}(t) + \kappa(t)v^2 \mathbf{N}(t)$, where $v = |\mathbf{r}'(t)|$, the speed at time t .)

3. Let $\mathbf{B}(t) = \mathbf{r}'(t) \times \mathbf{r}''(t) / |\mathbf{r}'(t) \times \mathbf{r}''(t)|$, the binormal to the curve $\mathbf{r}(t)$ in \mathbb{R}^3 . Suppose that $\mathbf{B}(t) = \mathbf{n}$ for all t , where $\mathbf{n} = (a,b,c)$ is some fixed vector in \mathbb{R}^3 of length 1. Show that $\mathbf{r}(t)$ lies entirely in a plane with normal \mathbf{n} .

(Hint : Let $f(t) = \mathbf{r}(t) \cdot \mathbf{n}$ (dot product). Show that $f(t)$ is a constant function by showing that $f'(t) = 0$ for all t . Then apply the result of extra problem 1.)

4. Let $x = a + bi + cj + dk$ and $y = A + Bi + Cj + Dk$ be any two quaternions. Derive each of the following facts :

a) $\operatorname{Re} \overline{xy} = x \cdot y$ (dot product) and $\operatorname{Re} xy = \operatorname{Re} yx$

b) $\overline{xy} = \overline{y} \overline{x}$ (note that the multiplication order is reversed)

c) $x \overline{x} = \overline{xx} = |x|^2 = a^2 + b^2 + c^2 + d^2$

5. a) Prove that every real number A has the property that $xA = Ax$ for every quaternion $x = a + bi + cj + dk$.

b) Let $y = A + Bi + Cj + Dk$ be a quaternion with the property that $yx = xy$ for all quaternions x . Prove that $y = A$ is a real number (i.e. $B = C = D = 0$).