§ 10.3 # 11, 13, 15, 19, 21
10.4 # 10, 11, 20, 23

§ 10.3 # 11) $r(t) = (2\sin t, 5t, 2\cos t)$

$r'(t) = (2\cos t, 5, -2\sin t)$

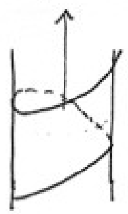
$|r'(t)| = \{4\cos^2 t + 25 + 4\sin^2 t\}^{1/2} = \sqrt{29}$

$$\boxed{T(t) = \frac{r'(t)}{|r'(t)|} = \left(\frac{2}{\sqrt{29}}\cos t, \frac{5}{\sqrt{29}}, \frac{-2}{\sqrt{29}}\sin t\right)}$$

$$T'(t) = \left(\frac{-2}{\sqrt{29}}\sin t, 0, \frac{-2}{\sqrt{29}}\cos t\right)$$

$$|T'(t)| = \frac{2}{\sqrt{29}} \Rightarrow \boxed{N(t) = \frac{T'(t)}{|T'(t)|} = (-\sin t, 0, -\cos t)}$$

b) $$K(t) = \frac{|T'(t)|}{|r'(t)|} = \frac{2/\sqrt{29}}{\sqrt{29}} = \boxed{\frac{2}{29}}$$



This curve is a helix on a cylinder whose central axis is the x-axis

13) $r(t) = (\sqrt{2}t, e^t, e^{-t})$

$r'(t) = (\sqrt{2}, e^t, -e^{-t})$, $|r'(t)| = \{2 + e^{2t} + e^{-2t}\}^{1/2}$
$= \{(e^t + e^{-t})^2\}^{1/2}$
$= e^t + e^{-t}$
$= \dfrac{e^{2t}+1}{e^t}$

$$T(t) = \frac{r'(t)}{|r'(t)|} = \frac{e^t}{1+e^{2t}}(\sqrt{2}, e^t, -e^{-t})$$

$$\boxed{T(t) = \left(\frac{\sqrt{2}e^t}{1+e^{2t}}, \frac{e^{2t}}{1+e^{2t}}, \frac{-1}{1+e^{2t}}\right)}$$

$$T'(t) = \left(\frac{\sqrt{2}e^t - \sqrt{2}e^{3t}}{(1+e^{2t})^2}, \frac{2e^{2t}}{(1+e^{2t})^2}, \frac{2e^{2t}}{(1+e^{2t})^2}\right)$$

$$|T'(t)| = \left\{\frac{2e^{2t}}{(1+e^{2t})^2}\right\}^{1/2} = \frac{\sqrt{2}e^t}{1+e^{2t}}$$ (details omitted)

$$\boxed{N(t) = \frac{T'(t)}{|T'(t)|} = \left(\frac{1-e^{2t}}{1+e^{2t}}, \frac{\sqrt{2}e^t}{1+e^{2t}}, \frac{\sqrt{2}e^t}{1+e^{2t}}\right)}$$

$$\boxed{K(t) = \frac{|T'(t)|}{|r'(t)|} = \frac{\sqrt{2}e^t / 1+e^{2t}}{(1+e^{2t})/e^t} = \frac{\sqrt{2}e^{2t}}{(1+e^{2t})^2}}$$

15) $r(t) = t^2 i + t k = (t^2, 0, t)$

$$K(t) = \frac{|r' \times r''|}{|r'|^3}$$

$r' = (2t, 0, 1) \qquad r'' = (2, 0, 0)$

$$r' \times r'' = \det \begin{pmatrix} i & j & k \\ 2t & 0 & 1 \\ 2 & 0 & 0 \end{pmatrix}$$

$= i(0) \quad -j(-2) + k(0) = (0, 2, 0)$

$|r' \times r''| = 2 \quad , \quad |r'|^3 = (1+4t^2)^{3/2}$

$$\boxed{K(t) = \frac{2}{(1+4t^2)^{3/2}}}$$

19) $r(t) = (t, t^2, t^3) \quad , \quad r'(t) = (1, 2t, 3t^2)$

$r''(t) = (0, 2, 6t) \quad , \quad r' \times r'' = (6t^2, -6t, 2)$

$|r' \times r''| = (36t^4 + 36t^2 + 4)^{1/2}$

$|r'| = (1 + 4t^2 + 9t^4)^{1/2}$

$$K(t) = \frac{(4 + 36t^2 + 36t^4)^{1/2}}{(1+4t^2+9t^4)^{3/2}}$$

at $t = 1$, $K(1) = \dfrac{(76)^{1/2}}{(14)^{3/2}} = \dfrac{2\sqrt{19}}{14\sqrt{14}} = \dfrac{1}{7}\sqrt{\dfrac{19}{14}}$

21) $y = xe^x \qquad K = \dfrac{|y''|}{\{1+(y')^2\}^{3/2}}$

$y' = e^x + xe^x = (1+x)e^x$

$y'' = e^x + e^x + xe^x = 2e^x + xe^x = e^x(x+2)$

$1 + (y')^2 = 1 + (1+x)^2 e^{2x}$

$$K = \frac{e^x |x+2|}{\{1 + (1+x)^2 e^{2x}\}^{3/2}}$$

§10.4 # 10) $r(t) = (2\cos t, 3t, 2\sin t)$

$r' = (-2\sin t, 3, 2\cos t)$ velocity

$r'' = (-2\cos t, 0, -2\sin t)$ acceleration

$|r'(t)| = \{4\sin^2 t + 9 + 4\cos^2 t\}^{1/2} = \sqrt{13}$ speed

11) $r(t) = (\sqrt{2}t, e^t, e^{-t})$

$r'(t) = (\sqrt{2}, e^t, -e^{-t})$ velocity

$r''(t) = (0, e^t, e^{-t})$ acceleration

$|r'(t)| = (2 + e^{2t} + e^{-2t})^{1/2} = \{(e^t + e^{-t})^2\}^{1/2}$

$= e^t + e^{-t}$ speed

20) By hypothesis $|r'(t)| = c$, c a constant

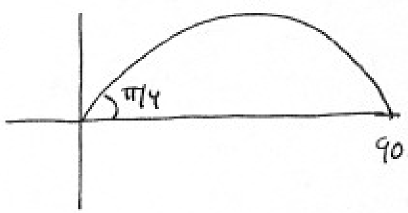
$\Rightarrow c^2 = |r'(t)| = r'(t) \cdot r'(t)$ differentiate with respect to t

$\frac{d}{dt}$: $0 = r''(t) \cdot r'(t) + r'(t) \cdot r''(t)$

$= 2r' \cdot r'' \Rightarrow \boxed{0 = r' \cdot r''}$

Since r' is the velocity and r'' is the acceleration, the boxed equation above says that the velocity is orthogonal to the acceleration.

23)



From class or from the book we have for a projectile launched with elevation angle θ

$r(t) = r(0) + tV_0 + (0, -\frac{1}{2}gt^2)$

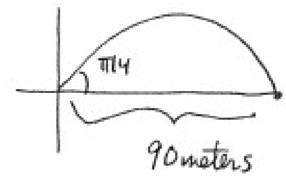
$= r(0) + t\,(|V_0|\cos\theta, |V_0|\sin\theta) + (0, -\frac{1}{2}gt^2)$

If $r(0) = (0,0)$ as in this case, then

$x(t) = t\,|V_0|\cos\theta = t\,|V_0|\frac{\sqrt{2}}{2}$

$y(t) = t\,|V_0|\sin\theta - \frac{1}{2}gt^2 = t|V_0|\frac{\sqrt{2}}{2} - 4.9t^2$

Here we are given $\theta = \pi/4$ and distance is in meters so $g = 9.8 \frac{\text{meters}}{\text{sec}^2}$



Let t_0 be the time the projectile takes to hit the ground.

Method 1 Then $90 = x(t_0) = t_0|V_0|\frac{\sqrt{2}}{2}$ (1)

$0 = y(t_0) = t_0|V_0|\frac{\sqrt{2}}{2} - (4.9)t_0^2$ (2)

Subtracting these two equations yields

$90 = (4.9)t_0^2 \Rightarrow t_0 = \sqrt{\frac{90}{4.9}} = \sqrt{\frac{900}{49}}$

$\boxed{t_0 = \frac{30}{7} \text{ seconds}}$

From the first equation above we have

$90 = (\frac{30}{7})|V_0|\frac{\sqrt{2}}{2} \Rightarrow \boxed{|V_0| = 21\sqrt{2} \cong 29.68 \text{ meters/sec}}$

Method 2 $0 = t_0(|V_0|\frac{\sqrt{2}}{2} - 4.9t_0)$ equation two

$\Rightarrow t_0 = \frac{|V_0|\sqrt{2}}{9.8}$. Now plug into equation (1), solve for $|V_0|$.