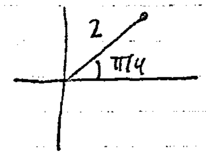
Math 33A HW #4 Answer Sheet

§ 9.7 # 3a, 4a, 5a, 6a, 7a, 9a, 11, 12, 13, 15, 18, 21

10.2 # 9, 11, 15, 16, 19, 20

Two extra problems

§ 9.7 # 3a) $(2, \pi/4, 1) = (r, \theta, z)$

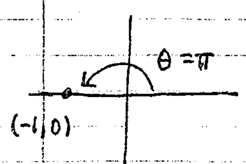


$x = r\cos\theta = 2\cos(\pi/4) = \sqrt{2}$

$y = r\sin\theta = 2\sin(\pi/4) = \sqrt{2}$

$\boxed{\text{Answer } (\sqrt{2}, \sqrt{2}, 1)}$

4a) $(1, \pi, e)$ By inspection: $(-1, 0, e)$



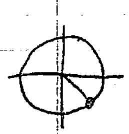
Check $x = r\cos\theta = (1)\cos(\pi) = -1$

$y = r\sin\theta = (1)\sin(\pi) = 0$

5a) $(1, -1, 4) = (x, y, z)$

$r = (x^2+y^2)^{1/2} = (1+1)^{1/2} = \sqrt{2}$

$\cos\theta = \frac{x}{r} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$, $\sin\theta = \frac{y}{r} = \frac{-1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$



$x > 0$ and $y < 0$ $\Rightarrow$ fourth quadrant

$\theta = 2\pi - \pi/4 = 7\pi/4 \Rightarrow \boxed{(\sqrt{2}, 7\pi/4, 4)}$

Math 33A HW #4 Answer Sheet

6a) $(3, 3, -2) = (x, y, z)$

$r = (x^2+y^2)^{1/2} = (18)^{1/2} = 3\sqrt{2}$

$\cos\theta = \frac{x}{r} = \frac{3}{3\sqrt{2}} = \frac{\sqrt{2}}{2}$, $\sin\theta = \frac{y}{r} = \frac{3}{3\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$x > 0, y > 0 \Rightarrow$ first quadrant $\Rightarrow \theta = \pi/4$

$\boxed{\text{Answer } (r, \theta, z) = (3\sqrt{2}, \pi/4, -2)}$

7a) $(1, 0, 0) = (\rho, \theta, \varphi)$

$z = \rho\cos\varphi = (1)\cos(0) = 1$

$r = \rho\sin\varphi = (1)\sin\theta = 0$

$x = r\cos\theta = 0$, $y = r\sin\theta = 0$

$\boxed{\text{Answer } (0, 0, 1) = (x, y, z)}$

9a) $(1, \sqrt{3}, 2\sqrt{3}) = (x, y, z)$

$\rho = (x^2+y^2+z^2)^{1/2} = (1 + 3 + 12) = 4$

$\cos\varphi = \frac{z}{\rho} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} \Rightarrow \boxed{\varphi = \pi/6}$

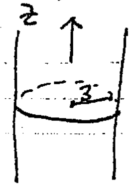
$r = (x^2+y^2)^{1/2} = (1+3)^{1/2} = 2$

$\cos\theta = \frac{x}{r} = \frac{1}{2}$; $\sin\theta = \frac{y}{r} = \frac{\sqrt{3}}{2}$

first quadrant $\Rightarrow \theta = \pi/3 = 60^\circ$

$\boxed{\text{Answer} \quad (\rho, \theta, \varphi) = (4, \pi/3, \pi/6)}$

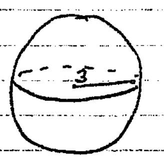
11) $r = 3$, $9 = r^2 = x^2+y^2$



cylinder whose axis is the z-axis

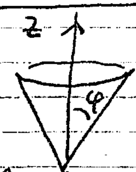
radius = 3

(12) $\rho = 3 \Rightarrow 9 = \rho^2 = x^2+y^2+z^2$



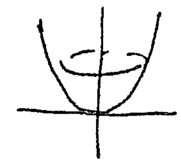
sphere of radius 3 centered at (0,0,0)

13) $\phi = \pi/3$



The surface is a half cone with angle $\pi/3$ as shown

(15) $z = r^2 = x^2+y^2$



nose cone – this description and picture is good enough for me

(18) $\rho = 2\cos\varphi = \frac{2z}{\rho} \Rightarrow 2z = \rho^2 = x^2+y^2 + z^2$

$\Rightarrow 0 = x^2+y^2+z^2-2z \Rightarrow 1 = x^2+y^2+(z^2-2z+1)$

$\Rightarrow 1 = x^2+y^2+(z-1)^2$

$\boxed{\text{Sphere of radius 1 centered at } (0,0,1)}$

21) $z = x^2+y^2$

a) cylindrical coordinates: $z = r^2$

b) spherical coordinates : $\rho\cos\varphi = r^2 = (\rho\sin\varphi)^2$

$= \rho^2\sin^2\varphi \Rightarrow \boxed{\cos\varphi = \rho\sin^2\varphi}$

§ 10.2 # 9) $r(t) = (t^2, 1-t, t^{1/2})$

$r'(t) = (2t, -1, \frac{1}{2}t^{-1/2}) = (2t, -1, \frac{1}{2\sqrt{t}})$

11) $r(t) = (e^{t^2}, -1, \ln(1+3t))$

$r'(t) = (2te^{t^2}, 0, \frac{3}{1+3t})$

$= (2te^{t^2})i + (\frac{3}{1+3t})k$

(15) $r(t) = (\cos t, 3t, 2\sin(2t))$

$r'(t) = (-\sin t, 3, 4\cos(2t))$

$|r'(t)| = \{\sin^2 t + 9 + 16\cos^2(2t)\}^{1/2}$

$T(t) = \frac{r'(t)}{|r'(t)|} = \frac{(-\sin t, 3, 4\cos(2t))}{\{\sin^2 t + 9 + 16\cos^2(2t)\}^{1/2}}$

when $t=0$, $T(0) = \frac{(0, 3, 4)}{\{9+16\}^{1/2}} = (0, \frac{3}{5}, \frac{4}{5})$

$= (\frac{3}{5})j + (\frac{4}{5})k$

16) $r(t) = (2\sin t, 2\cos t, \tan t)$

$r'(t) = (2\cos t, -2\sin t, \sec^2 t)$

$|r'(t)| = \{4\cos^2 t + 4\sin^2 t + \sec^4 t\}^{1/2}$

$= \{4 + \sec^4 t\}^{1/2}$

$T(t) = \frac{r'(t)}{|r'(t)|} = \frac{(2\cos t, -2\sin t, \sec^2 t)}{\{4 + \sec^4 t\}^{1/2}}$

When $t = \pi/4$, $T(\pi/4) = \frac{(\sqrt{2}, -\sqrt{2}, 2)}{\{4+4\}^{1/2}}$

$= \frac{(\sqrt{2}, -\sqrt{2}, 2)}{2\sqrt{2}} = (\frac{1}{2}, -\frac{1}{2}, \frac{\sqrt{2}}{2})$

19) $r(t) = (t^5, t^4, t^3)$

$r'(t) = (5t^4, 4t^3, 3t^2)$

(1, 1, 1) occurs when $t = 1$

tangent line : $r(1) + t\,r'(1) = (1,1,1) + t(5,4,3)$

$= (1+5t, 1+4t, 1+3t)$

$\boxed{x = 1+5t,\ y = 1+4t,\ z = 1+3t}$ parametric equat of the tangent line

20) $r(t) = (t^2-1, t^2+1, t+1)$; $(-1, 1, 1)$

$(-1, 1, 1)$ occurs when $t=0$ $r'(t) = (2t, 2t, 1)$

tangent line: $r(0) + t\,r'(0) = (-1,1,1) + t(0,0,1)$

$= (-1, 1, 1+t)$ $\boxed{x = -1,\ y = 1,\ z = 1+t}$ parametric equations

Extra problem 1

General theory $z = ax^2 + bxy + cy^2 + dx + ey + f$

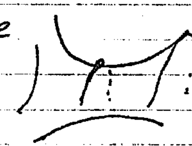
Let $D = 4ac - b^2$

Case 1 $D > 0,\ a > 0 \Rightarrow$

Case 2 $D > 0,\ a < 0 \Rightarrow$

Case 3 $D < 0$ Saddle surface

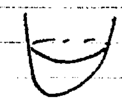
Case 4 $D = 0$ degenerate case



a) $z = 5x^2 + 2xy + 3y^2 + 4x - 2y + 6$

$D = 4ac - b^2 = 4(5)(3) - (2)^2 = 60 - 4 = 56 > 0$

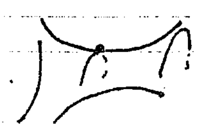
$a = 5 > 0 \Rightarrow$ Case 1



b) $z = 5x^2 + 10xy + 2y^2 + 2x + 5y + 9$

$D = 4ac - b^2 = 4(5)(2) - (10)^2 = 40 - 100 = -60 < 0$

$D < 0 \Rightarrow$ Saddle surface



Extra Problem 2 Let A be a 3×3 matrix such that $Ax \cdot Ay = x \cdot y$ (dot product) for all x, y in $\mathbb{R}^3$, where x, y are 3×1 matrices = column vectors

a) If $|x| = 1$, then $1 = |x|^2 = x \cdot x$

$\Rightarrow 1 = (Ax) \cdot (Ax) = |Ax|^2$ by hypothesis

$\Rightarrow |Ax| = 1$ also.

b) Let $\theta = \measuredangle(x, y)$ and $\alpha = \measuredangle(Ax, Ay)$

Then $\cos\theta = \dfrac{x \cdot y}{|x||y|}$ and $\cos\alpha = \dfrac{Ax \cdot Ay}{|Ax||Ay|}$

$= \dfrac{x \cdot y}{|Ax||Ay|}$ by hypothesis. It suffices to show $\alpha =$

$|Ax| = |x|$ and $|Ay| = |y|$ for then $\cos\theta = \cos\alpha \Rightarrow \theta$ QED

$|Ax|^2 = (Ax) \cdot (Ax) = x \cdot x = |x|^2$ for all x

c) $(A^t x) \cdot (A^t y) = x \cdot (AA^t y)$ HW #3 problem 2, since $(A^t)^t =$

$= x \cdot y$ by problem 3, HW #3

(Actually, this shows that $A^tA = Id$ but $AA^t = Id$ also)