

Homeomorphic Bernoulli measures on Cantor spaces.

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Abstract

Consider an experiment with k possible outcomes with probabilities r_1, r_2, \dots, r_k . Let $m(r_1, \dots, r_k)$ be the probability measure induced by conducting infinitely many Bernoulli trials. This measure is a product measure on X , the symbol space or Cantor space consisting of all infinite words on k letters. We consider the problem of when two such product measures are homeomorphic. So, given measures $m = m(r_1, \dots, r_k)$ and $w = m(s_1, \dots, s_k)$ when is there a homeomorphism h of X onto itself such that $m(E) = w(h(E))$ for each Borel set E ? This problem has its origins in a 1941 paper of Ulam and Oxtoby. There are many connections to number theory and ergodic theory.